Integration Test Review

1. $\int 2x^3 - 5x + 2\, dx$

$\frac{1}{2}x^4 - \frac{5}{2}x^2 + 2x + C$

2. $\int \frac{4x^3 + 5x^2 + 8x}{x}\, dx$

$\int 4x^2 + 5x + 8\, dx$

$\frac{4}{3}x^3 + \frac{5}{2}x^2 + 8x + C$

3. $\int 3x\sqrt{x} + 2\, dx$

$\int 3x^{3/2} + 2\, dx$

$\frac{6}{5}x^{5/2} + 2x + C$

4. $\int \frac{5x^3 + 4x^{2/3} - 2x\sqrt[3]{x}}{3x^{1/2}}\, dx$

$\frac{1}{3}\int \left(5x^3 + 4x^{2/3} - 2x^{4/3}\right) x^{-1/2}\, dx$

$\frac{1}{3}\int 5x^{5/2} + 4x^{1/6} - 2x^{5/6}\, dx$

$\frac{1}{3}\left[\frac{10}{7}x^{7/2} + \frac{24}{7}x^{7/6} - \frac{12}{11}x^{11/6} + C\right]$

$\frac{10}{21}x^{7/2} + \frac{8}{7}x^{7/6} - \frac{4}{11}x^{11/6} + C$

5. $\int \tan x\, dx$

$-\ln|\cos x| + C$

8. $\int e^{4x-1}\, dx$

$u = 4x - 1$ $\quad \int e^u \cdot \frac{du}{4}$

$\frac{du}{dx} = 4$ $\quad \frac{1}{4}\int e^u\, du$

$dx = \frac{du}{4}$ $\quad \frac{1}{4}e^u + C$

$\frac{1}{4}e^{4x-1} + C$

7. $\int 2e^x\, dx$

$2e^x + C$

6. $\int \sqrt{3x+5}\, dx$

$u = 3x + 5$ $\quad \int u^{1/2} \cdot \frac{du}{3}$

$\frac{du}{dx} = 3$ $\quad \frac{1}{3}\int u^{1/2}\, du$

$dx = \frac{du}{3}$ $\quad \frac{2}{9}u^{3/2} + C$

$\frac{2}{9}(3x+5)^{3/2} + C$

9. $\int \cos^2(3x)\sin(3x)\, dx$

$u = \cos(3x)$ $\quad \int u^2 \cdot \cancel{\sin(3x)} \cdot \frac{du}{-3\cancel{\sin(3x)}}$

$\frac{du}{dx} = -\sin(3x) \cdot 3$ $\quad -\frac{1}{3}\int u^2\, du$

$dx = \frac{du}{-3\sin(3x)}$ $\quad -\frac{1}{9}u^3 + C$

$-\frac{1}{9}\cos^3(3x) + C$

10. $\int 5x\sqrt{3x^2+5}\, dx$

$u = 3x^2 + 5$ $\quad \int 5\cancel{x} u^{1/2} \cdot \frac{du}{6\cancel{x}}$

$\frac{du}{dx} = 6x$ $\quad \frac{5}{6}\int u^{1/2}\, du$

$dx = \frac{du}{6x}$ $\quad \frac{5}{9}u^{3/2} + C$

$\frac{5}{9}(3x^2+5)^{3/2} + C$

11. $\int_0^3 (2x^3 - 5)\, dx$

$\frac{1}{2}x^4 - 5x \Big|_0^3$

$\left[\frac{1}{2}(3)^4 - 5(3)\right] - [0]$

$\frac{81}{2} - 15$

$\frac{51}{2}$

12. $\int_0^{\pi} \tan x\, dx$

$-\ln|\cos x| \Big|_0^{\pi}$

$-\ln|\cos \pi| + \ln|\cos(0)|$

$0 + 0$

0

13. $\int_2^5 x\sqrt{x^2+3}\,dx$

$u = x^2+3$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

bounds: $(2)^2+3 = 7$
$(5)^2+3 = 28$

$\int_7^{28} \cancel{x}\cdot u^{1/2}\cdot \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int_7^{28} u^{1/2}\,du$

$\frac{1}{3}u^{3/2}\Big|_7^{28}$

$\frac{1}{3}(28)^{3/2} - \frac{1}{3}(7)^{3/2}$

14. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin(2x)\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

bounds: $2(\frac{\pi}{2}) = \pi$
$2(\frac{3\pi}{2}) = 3\pi$

$\int_{\pi}^{3\pi} \sin u \cdot \frac{du}{2}$

$\frac{1}{2}\int_{\pi}^{3\pi} \sin u\,du$

$-\frac{1}{2}\cos u \Big|_{\pi}^{3\pi}$

$-\frac{1}{2}\left[\cos(3\pi) - \cos(\pi)\right]$

$-\frac{1}{2}\left[-1 + (+1)\right]$

0

15. $f(x) = x^3-3$ $\quad [2,4]$ $\quad n=6$

$\Delta x = \frac{4-2}{6} = \frac{1}{3}$ $\quad x_0 = 2$ $\quad x_1 = \frac{7}{3}$ $\quad x_2 = \frac{8}{3}$ $\quad x_3 = 3$ $\quad x_4 = \frac{10}{3}$ $\quad x_5 = \frac{11}{3}$ $\quad x_6 = 4$

a) $R_6 = (\frac{1}{3})\left[f(\frac{7}{3}) + f(\frac{8}{3}) + f(3) + f(\frac{10}{3}) + f(\frac{11}{3}) + f(4)\right]$

$R_6 = 14.704$ units^2

b) $L_6 = (\frac{1}{3})\left[f(2) + f(\frac{7}{3}) + f(\frac{8}{3}) + f(3) + f(\frac{10}{3}) + f(\frac{11}{3})\right]$

$L_6 = 10.704$ units^2

c) $M_6 = (\frac{1}{3})\left[f(\frac{13}{6}) + f(\frac{15}{6}) + f(\frac{17}{6}) + f(\frac{19}{6}) + f(\frac{21}{6})\right]$

$M_6 = 12.648$ units^2

d) $T_6 = (\frac{1}{6})\left[f(2) + 2f(\frac{7}{3}) + 2f(\frac{8}{3}) + 2f(3) + 2f(\frac{10}{3}) + 2f(\frac{11}{3}) + f(4)\right]$

$T_6 = 12.704$ units^2

16. $f(x) = \sqrt{x} - 1 \quad [1,9] \quad n=4$

$\Delta x = \frac{9-1}{4} = 2 \quad x_0 = 1 \quad x_1 = 3 \quad x_2 = 5 \quad x_3 = 7 \quad x_4 = 9$

a) $R_4 = (2)[f(3) + f(5) + f(7) + f(9)]$

$R_4 = 11.228$ units^2

b) $L_4 = (2)[f(1) + f(3) + f(5) + f(7)]$

$L_4 = 7.228$ units^2

c) $M_4 = (2)[f(2) + f(4) + f(6) + f(8)]$

$M_4 = 9.384$ units^2

d) $T_4 = (1)[f(1) + 2f(3) + 2f(5) + 2f(7) + f(9)]$

$T_4 = 9.228$ units^2

17. $f(x) = x^3 + x^2 - 6x \quad [0,3]$

$0 = x(x^2 + x - 6)$

$0 = x(x+3)(x-2)$

$x = 0 \quad \cancel{x = -3} \quad x = 2$

$\left|\int_0^2 x^3 + x^2 - 6x\,dx\right| + \int_2^3 x^3 + x^2 - 6x\,dx$

$\left|\frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 \Big|_0^2\right| + \frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 \Big|_2^3$

$\left|\frac{-16}{3}\right| + \frac{91}{12} = \frac{155}{12}$ units^2

18. $f(x) = \sqrt{2x-5} - 3 \quad [3,15]$

$0 = \sqrt{2x-5} - 3$

$3 = \sqrt{2x-5}$

$9 = 2x - 5$

$14 = 2x$

$x = 7$

$\left|\int_3^7 \sqrt{2x-5} - 3\,dx\right| + \int_7^{15} \sqrt{2x-5} - 3\,dx$

$\left|\frac{-10}{3}\right| + \left|\frac{26}{3}\right| = 12$ units^2

19. $f(x) = \frac{3}{x} \quad [e, e^2]$

$3\int_e^{e^2} \frac{1}{x}\,dx$

$3\ln|x| \Big|_e^{e^2}$

$3[\ln e^2 - \ln e]$

3 units^2

20. $f(x) = \cos x \quad [0, \frac{3\pi}{2}]$

$\cos x = 0$

$x = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \cos x \, dx + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x \, dx \right|$$

$$\sin x \Big|_0^{\frac{\pi}{2}} + \left| \sin x \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \right|$$

$$\left[\sin\tfrac{\pi}{2} - \sin 0\right] + \left| \left[\sin\tfrac{3\pi}{2} - \sin\tfrac{\pi}{2}\right] \right|$$

$$1 + |-2| = 3 \text{ units}^2$$

21. $v(t) = 3t^2 - 2t - 8 \quad [0, 4]$

a) $(3t+4)(t-2)$

$\cancel{t = -\frac{4}{3}} \qquad t = 2 \text{ sec}$

$$\left| \int_0^2 3t^2 - 2t - 8 \, dt \right| + \int_2^4 3t^2 - 2t - 8 \, dt$$

$$\left| t^3 - t^2 - 8t \Big|_0^2 \right| + t^3 - t^2 - 8t \Big|_2^4$$

$$\left| \left[(2)^3 - (2)^2 - 8(2)\right] - [0] \right| + \left[(4)^3 - (4)^2 - 8(4)\right] - [-12]$$

$$|-12| + 28$$

$$40 \text{ ft.}$$

b) $\int_0^4 3t^2 - 2t - 8 \, dt$

$t^3 - t^2 - 8t \Big|_0^4$

$[16] - [0]$

16 ft

c) $a(3) = v'(3)$

$v'(t) = 6t - 2$

$v'(3) = 6(3) - 2$

16 ft/se

d) $\left| \int_0^2 3t^2 - 2t - 8 \, dt \right| + \int_2^3 3t^2 - 2t - 8 \, dt$

$$\left| t^3 - t^2 - 8t \Big|_0^2 \right| + t^3 - t^2 - 8t \Big|_2^3$$

$$|-12| + \left[(3)^3 - (3)^2 - 8(3)\right] - [-12]$$

$$12 + 6$$

$$18 \text{ ft.}$$

e) $\int_0^3 3t^2-2t-8\ dt$

$t^3-t^2-8t\ \Big|_0^3$

$[-6]-[0]$

-6 ft.

f) $a(t)=6t-2$

$a(1)=6(1)-2$

4 ft/sec^2

g) $\frac{1}{4-0}\int_0^4 3t^2-2t-8\ dt$

$\frac{1}{4}\left[t^3-t^2-8t\ \Big|_0^4\right]$

$\frac{1}{4}[16-0]$

4 ft/sec

22. $v(t)=t^3-3t^2-4t \qquad [0,5]$

a) $0=t(t^2-3t-4)$

$0=t(t-4)(t+1)$

$t=0 \quad t=4 \quad t=-1$

$\left|\int_0^4 t^3-3t^2-4t\ dt\right| + \int_4^5 t^3-3t^2-4t\ dt$

$\left|\frac{1}{4}t^4-t^3-2t^2\ \Big|_0^4\right| + \frac{1}{4}t^4-t^3-2t^2\ \Big|_4^5$

$\left|[64-64-32]-[0]\right| + \left[\frac{1}{4}(5)^4-(5)^3-2(5)^2\right]-[-32]$

$|-32| + \left[\frac{625}{4}-125-50\right]+32$

$32 + 13.25$

45.25 ft.

b) $\int_0^5 t^3-3t^2-4t\ dt$

$\frac{1}{4}t^4-t^3-2t^2\ \Big|_0^5$

$\left[\frac{1}{4}(5)^4-(5)^3-2(5)^2\right]-[0]$

-18.75 ft.

c) $a(t) = v'(t) = 3t^2 - 6t - 4$

$a(2) = 3(2)^2 - 6(2) - 4$

$a(2) = -4$ ft/sec^2

d) $\left| \int_0^3 t^3 - 3t^2 - 4t \, dt \right|$

$\left| \frac{1}{4}t^4 - t^3 - 2t^2 \Big|_0^3 \right|$

$\left| \left[\frac{1}{4}(3)^4 - (3)^3 - 2(3)^2\right] - [0] \right|$

$|-24.75|$

24.75 ft.

e) $\int_0^3 t^3 - 3t^2 - 4t \, dt$

-24.75 ft.

f) $a(4) = 3(4)^2 - 6(4) - 4$

$a(4) = 20$ ft/sec^2

g) $s(t) = \int t^3 - 3t^2 - 4t \, dt$

$s(t) = \frac{1}{4}t^4 - t^3 - 2t^2 + C$

$(-17) = C$

$s(t) = \frac{1}{4}t^4 - t^3 - 2t^2 - 17$

$S(5) = \frac{1}{4}(5)^4 - (5)^3 - 2(5)^2 - 17$

$S(5) = \frac{625}{4} - 125 - 50 - 17$

$S(5) = -35.75$ ft.

h) $\frac{s(4.5) - s(0)}{4.5 - 0} = -6.469$ ft/sec

23. a) $r(30) = 36.508$ people/min

b) $\int_0^{40} r(t)\,dt + 57 = 1057.219$ people

c) $\frac{1}{40-25}\int_{25}^{40} r(t)\,dt = 39.384$ people/min

24. $\frac{dy}{dx} = 6x^2y - 3y$

$\frac{dy}{dx} = 3y(2x^2-1)$

$dy = 3y(2x^2-1)\,dx$

$\int \frac{1}{3y}\,dy = \int (2x^2-1)\,dx$

$\frac{1}{3}\ln|y| = \frac{2}{3}x^3 - x + C$

$\frac{1}{3}\ln|-4| = \frac{2}{3}(2)^3 - (2) + C$

$\frac{1}{3}\ln 4 = \frac{16}{3} - 2 + C$

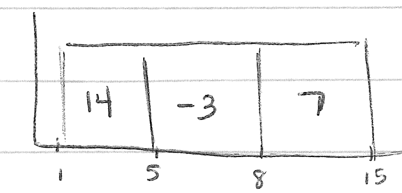
$\frac{1}{3}\ln 4 = \frac{10}{3} + C$

$\frac{1}{3}\ln 4 - \frac{10}{3} = C$

$3\left(\frac{1}{3}\ln|y| = \frac{2}{3}x^3 - x + \frac{1}{3}\ln 4 - \frac{10}{3}\right)$

$\ln|y| = 2x^3 - 3x + \ln 4 - 10$

$y = e^{2x^3-3x+\ln 4-10}$



28. $\int_1^5 f(x)\,dx = 14$

29. $\int_{15}^1 f(x)\,dx = -18$

30. $\int_5^{15} 2f(x) = 8$

31. $\int_{15}^8 f(x)\,dx = -7$

25. $\frac{d}{dx}\int_3^{x^3} \tan(2x)\,dx$

$[\tan(2(x^3))]\,3x^2$

$3x^2\tan(2x^3)$

26. $\frac{d}{dx}\int_5^{\sin(3x)} (x^2-5)\,dx$

$[(\sin 3x)^2 - 5]\cos(3x)\cdot 3$

27. $\frac{d}{dx}\int_{2x^2}^{3} (x^2-5x+1)\,dx = \frac{d}{dx} - \int_3^{2x^2} x^2-5x+1\,dx$

$-[(2x^2)^2 - 5(2x^2) + 1](4x)$

$-4x(4x^4 - 10x^2 + 1)$